AI-Powered Two-Phase Method for Microscopic Periodic Railway Operation Diagrams

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Abstract

In the actual organization of railroad transportation, the periodic train schedule of the railroad provides for the arrival, departure or passage of the train at each station in a fixed period, and these time points will be fixed and repeated in each period, so that the periodic train schedule has a significant predictability. This fully demonstrates the notable advantages of high-speed railways in terms of speed and comfort, allowing passengers to conveniently transfer at interchange stations within the railway network, achieving a 'public transit-like' operation. This paper conducts a microscopic modeling of the railway network based on track circuit sections, and constructs a 0-1 integer programming model for the optimization of periodic train timetable compilation using a time-discretized extended space-time network approach. The model is decomposed according to the solution idea of train decomposition by adopting a grouped sorting method, optimizing only the optimal space-time path of one train route at a time, and the sub-model is solved by calling commercial optimization software. An integer linear programming model is established using operations research optimization methods, and an efficient decomposition algorithm is designed to solve the model, effectively improving the utilization rate of railway line capacity and the quality of transportation services. It innovatively applies a time-discretized extended space-time network method, integrating artificial intelligence (AI) optimization algorithms to construct a 0-1 integer programming model for compiling periodic train timetables.

Index Terms— AI Optimization, Two-Phase Method, Microscopic Modeling, Time-Discretized Extended Space-Time Network

1 Introduction

Railways, as a green and economical mode of transportation, have become an indispensable component.[1]-[4] Although railway transportation has experienced rapid development in recent years due to its advantages of low transportation costs and high efficiency, and the volume of transportation has continued to increase, the rapid growth of China's economy has accelerated the production of various products and simultaneously increased the demand for goods transportation. Consequently, railway transportation often faces a situation where supply falls short of demand, significantly affecting the efficiency of passengers' normal travel and the timely delivery of goods.[5]-[9] Therefore, under certain infrastructure conditions, how to scientifically utilize transportation organization methods to maximize the efficiency of limited railway capacity has become the key to solving the problem, with related issues of the train schedule being particularly important. High-speed trains in Europe, Japan, and Taiwan have almost universally adopted periodic train schedules. In practical applications, the timetable for peak periods is usually compiled first, and then adjustments are made based on fluctuations in passenger flow, such as removing lines or fine-tuning some running lines to form timetables for other periods. The timetables for weekdays and weekends or different seasons are also adjusted accordingly. Lines organized under the periodic train timetable model have fully demonstrated their effectiveness through long-term operational experience.[10]-[17] Looking at China's high-speed rails, most lines already have the conditions to operate fully periodic train times, and the remaining lines can adopt a mixed structure of 'periodic + non-periodic' train times. However, even so, as China's high-speed railway network gradually expands and the number of passenger transports rapidly increases, research on periodic timetable issues in China is not comprehensive enough, and the rate of utilization of the railway capacity needs to be improved urgently. In the practical organization of railway transportation, the train timetable specifies the order in which trains occupy sections, the times at which trains depart, arrive, or pass through each station, the running times in sections, the stopping times at stations, as well as the weight and length of the trains.[18]-[24] The periodic railway timetable fixes the times of arrival, departure, or passing events at each station within a period, and these event times need to be repeated in each period. Therefore, the periodic train schedule has strong regularity, fully demonstrating the significant advantages of high-speed railways in terms of speed and comfort. Passengers can conveniently transfer to interchange stations within the rail network, achieving a convenient travel and ticketing model similar to the 'public transit' operation.[25]-[34]

Unlike traditional macroscopic train timetables, the finely crafted periodic train timetable falls under microscopic train scheduling. At this micro-level, the model considers track circuit sections as basic units and incorporates locking times as constraints, aiming to minimize the total train travel time across the network. AI algorithms here can predict the microscopic details of train operation, such as optimal acceleration and deceleration strategies under different signaling conditions, further optimizing running times and reducing energy consumption. Compared to headway-based running modes, this approach, through AI's precise control, can effectively avoid conflicts within sections, enhancing operational safety.

2 Literature review

Recent years have witnessed a surge of interest in the problem of train timetable construction, both in China and abroad. Numerous scholars and experts have conducted extensive research on this topic. Wang proposed a periodic potential difference model based on the fundamental cycle inequalities in the constraint graph of the CPF model, and investigated a periodic timetable construction model and algorithm using a periodic constraint graph. Li studied a railway scheduling model under temporary speed restrictions by refining the computation of train occupation times for block sections and incorporating speed constraints into the dispatching model. Nie generated peak-hour periodic train timetables using a breadth-first search approach and employed a depth-first search strategy to add non-periodic train services. Xie established a periodic train timetable model based on the Periodic Event Scheduling Problem (PESP), proposing a sequencing-based model tailored to the complex operations of Chinese high-speed passenger lines.NachtigallVoget addressed the periodic network optimization problem by minimizing passenger transfer waiting times and developed a genetic algorithm combining greedy heuristics and local improvement strategies, which was validated on a railway network with 26 lines and 37 stations. Other solution methods for periodic timetables include branch and bound techniques used by Zimmermann-Lindner, SAT solvers employed by Gattermann,[35]-[40] and the simplex method adopted by Nachtigall. Currently, non-periodic timetables - widely used in China's conventional railway lines - are often modeled using discrete time-space networks and the big M method, among others, which cover timetable optimization and adjustment problems. For periodic timetables, mainstream methods include the PESP model, the equivalent CPF model, and discrete time-space networks. However, research on micro-level optimization of periodic timetables remains limited. At the micro level, models take the track circuit segments as basic units and consider locking times as constraints, aiming to minimize the total travel time of trains across the network. Compared to the headway-based running mode, this approach can effectively avoid conflicts in block sections. Therefore, this study focuses on the detailed formulation of periodic train timetabling grounded in micro-level railway infrastructure characteristics. A discrete space-time network is constructed to represent the movement of trains along identical physical routes, which is subsequently extended through periodic expansion to form an augmented discrete space-time network. Based on this framework, a micro-level train scheduling optimization model is developed with the objective of minimizing the total travel time across the network. To solve the model efficiently, a two-phase decomposition approach is employed, wherein the overall problem is partitioned by individual trains. Each resulting subproblem optimizes the space-time trajectory of a single train line and is solved using commercial optimization software.[41]-[44]

3 Model formulation

3.1 Periodic runtime graph modeling framework based on extended spatiotemporal network

The finely crafted periodic train timetable, distinct from the traditional macroscopic train timetable, belongs to the microscopic level of train scheduling. As illustrated in 1, in a detailed periodic timetable, it is necessary to regard the station areas and the sections between stations as individual railway components, including nodes, switches, track circuits, and so on. The microscopic train timetable has the following fundamental requirements: a track circuit can only be occupied once at any given time, meaning only one train is allowed to pass through; the arrival and departure tracks within a station are used for trains to stop at the station, and trains are not permitted to stop on the main tracks; the station area has station boundaries, with boundary points serving as nodes that delineate the limits between the station area and the sections.

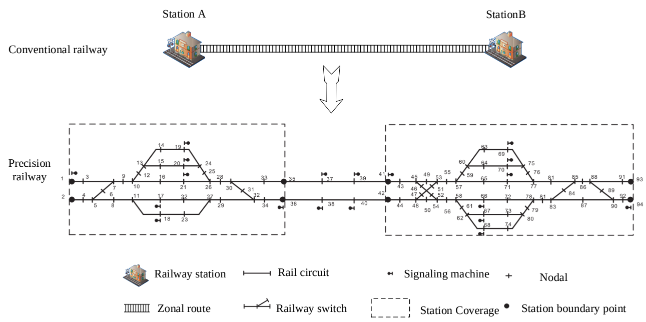


Figure 1: Comparison chart of traditional railways and fine railways

A route refers to the path a train takes from one location to another within a station. Each route, bounded by route nodes, includes two elements: track circuits and switches, and considering safety constraints, a route can only be occupied by one train at a time. As the starting and ending points of different routes vary, routes can be categorized into four types: departure routes, reception routes, through routes, and shunting routes, with the first three collectively referred to as train routes, as shown in 2.

In the microscopic train timetable, the concept of train route blocking time is defined based on the railway line's signaling, interlocking, and block conditions. The train route is taken as the smallest unit to set the blocking time. If the blocking times of two train routes overlap, it indicates a potential conflict between these two trains. 3 illustrates the detailed composition and calculation method of the train route blocking time.

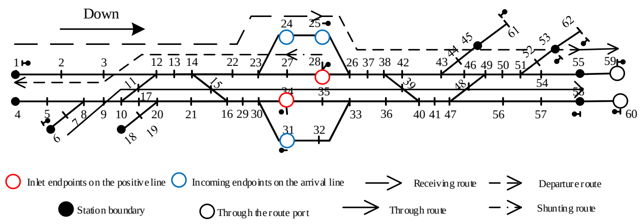


Figure 2: Schematic diagram of train approaches in a railway network at the micro level

Specifically, each track circuit group within the train route is used to calculate the process of train occupation of track resources, while each individual track circuit in the train route is utilized to calculate the train's running time within the route.

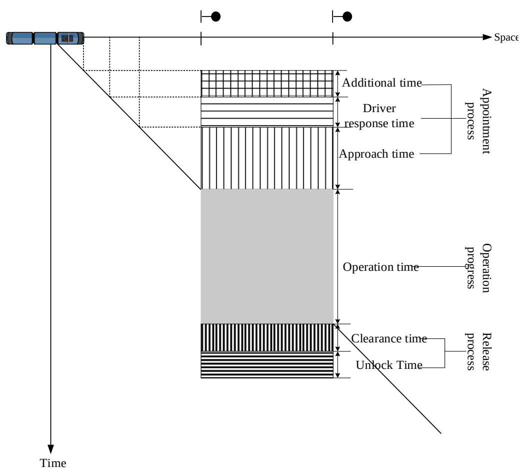


Figure 3: Schematic diagram of the locking time of the track section

The train route blocking time consists of three components: reservation time, running time, and release time. The reservation time includes the time required for setting up the signals and routes before the train enters the route, the driver's observation of the signal lights and reaction time, and the approach time from the indication signal to the entrance signal of the train route. The running time is the period from when the train's front end begins to enter the route (referred to as the train entry time) until the train's front end reaches the end of the route (referred to as the train exit time). The running time is calculated by summing up the running times on the track circuits that belong to the train route. It is assumed that the train can change direction by directly swapping the head and tail on the turnaround route, and thus the running time on the turnaround route can also be calculated based on this principle. The release time is the sum of the clearance time for the train's length and the route unlocking time. The minimum running time of the train on a track circuit is determined based on the length of the track circuit.

Table 1: Traction definition

Symbol	Definition
i, i', j, j'	Station index, $i, i', j, j' \in A$
(i, i')	Interval index, $(i, i') \in G$
l	Train scheme line index, $l \in L$
k, k'	Train index, $k, k' \in K$
t, t', t''	Discrete time unit index in the main space-time network, $t, t', t'' \in T'$
τ, τ', τ''	Extend discrete time unit index in spatiotemporal network, $\tau, \tau', \tau'' \in T''$
(i,t)	The index of the spatiotemporal nodes in the primary spatiotemporal network, $(i, t) \in V$
(i, i', t, t')	The spatiotemporal arc index in the main spatiotemporal network, $(i, i', t, t') \in E$
(i, τ)	Extend the spatiotemporal node index in the spatiotemporal network, $(i, \tau) \in V'$
(i, i', τ, τ')	Extend the spatiotemporal arc index in the spatiotemporal network, $(i, i', \tau, \tau') \in E'$

3.2 Discrete Space-Time Network Approach

To model the periodic train timetabling problem, the train operation plan for the section A-B-C-D over a time span of $H \cdot T$ is represented in a discrete space-time network. In this model, the movement and dwell of trains between stations are depicted as directed arrows (directed edges), as illustrated in 4 and 5, with the unit time length set to 1 minute. Here, A, B, C and D denote station names. Intermediate stations B and C are virtualized into two stations: B', B'' and C', C''. The first virtual station (B' or C') represents the 'arrival' at the station, while the second virtual station (B'' or C'') represents 'departure' from the station. If a train does not stop at the station, it directly proceeds to the second virtual station and departs from there, as exemplified by Train 1 passing through Station B. In contrast, if the train stops at the station, it first arrives at the first virtual station to wait and then departs from the second virtual station, as demonstrated by Train 1 passing through Station C.

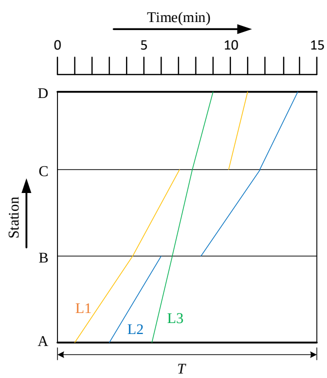


Figure 4: Regular periodic train timetable

Table 2: Collection definitions

Symbol	Definition
A	Station collection, including virtual stations
G	A collection of compartments, including virtual compartments
L	Train scheme line set
K	Train set
K_l	A collection of trains belonging to train scheme line l
V	A collection of space-time nodes in the main space-time network
V'	Extend the collection of spatiotemporal nodes in a spatiotemporal network
V_k	Extend the collection of spatiotemporal nodes in a spatiotemporal network
V'_k	A possible set of space-time nodes in an extended space-time network for train k
E	A collection of space-time arcs in the main space-time network
E'	Extend the collection of space-time arcs in a space-time network
E_k	The set of space-time arcs of train k in the main space-time network
E'_k	The set of spatiotemporal arcs of train k in an extended space-time network
T	A collection of discrete time units in a period, and the size of the set is the length of the period
T'	A collection of discrete time units in a primary space-time network
T''	Extend the collection of discrete time units in a spatiotemporal network
H	Periodic transformation coefficient of T
H'	Periodic transformation coefficient of T'
H''	Periodic transformation coefficient of T''
$\phi(j,j',\tau'')$	Extend the set of spatiotemporal arcs in the spatiotemporal network that are incompatible between interval (j,j') and time τ''

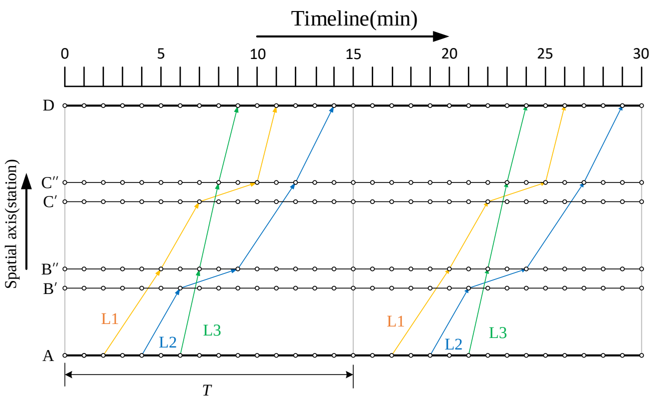


Figure 5: Periodic train diagram in a discrete space-time network

When modeling the train timetable optimization problem using the discrete space-time network approach, let V and E represent the sets of space-time nodes and space-time arcs in the discrete space-time network, respectively. Additionally, V_k and E_k denote the sets of space-time nodes and space-time arcs that train k may occupy. Furthermore, if $t, t', t'' \in T$ are used to index the discrete time units within the planning horizon T, then $(i,t) \in V$ and $(i,i',t,t') \in E'$ are used to index the sets V and E, respectively. Meanwhile, the set $\phi(j,j',t'')$ represents the set of space-time arcs in E that are mutually incompatible on section (j,j') at time t''.

$$\min Z = \sum_{k\in K} \sum_{(i,i',t,t')\in E_k} c_k(i,i',t,t') \cdot x_k(i,i',t,t') \quad (1)$$

$$\sum_{i,t:(i,i',t,t')\in E_k} x_k(i,i',t,t') - \sum_{i,t:(i',i,t',t)\in E_k} x_k(i',i,t',t) = \begin{cases} -1 & i'=o_k, t'=\text{dep}_k^s \\ 1 & i'=d_k, t'=T \\ 0 & \text{otherwise} \end{cases}, \quad \forall l \in L$$

$$\sum_{k\in K} \sum_{(i,i',t,t')\in\phi(j,j',t'')} x_k(i,i',t,t') \le 1, \ \forall (j,j') \in G, t'' \in T \quad (2)$$

$$x_k(i,i',t,t') \in \{0,1\}, \ \forall k \in K, (i,i',t,t') \in E \quad (3)$$

Equations 1 to 3 present a 0-1 integer programming model for optimizing the macroscopic non-periodic train timetable based on the discrete space-time network. The form of this model, apart from the objective function, is similar to that of the train timetable optimization model proposed by Caprara. The objective function in Equation 1 aims to minimize the total train operating cost, which can encompass various optimization objectives such as train travel time and energy consumption. In this section, the cost of using a space-time arc is set as the corresponding train's running time on that arc, making the objective function the minimization of total train travel time. Constraint corresponds to the flow balance relationship, ensuring that each train selects a unique space-time path. Specifically, Constraint uses the feasible space-time arc set E_k for train k, rather than the set E containing all space-time arcs. Therefore, by defining the set of space-time arcs that train k may traverse, the number of space-time arcs that need to be searched can be reduced, and unnecessary space-time arcs can be eliminated for each train k to meet dwell operation requirements. Constraint is the track capacity constraint, which

Table 3: Parameter definitions

Symbol	Definition
o_k	The station from which train *k* departs
d_k	The final station of train *k*
h_{dd}	The safety interval between two trains departing from the same station in the same direction, and the safety interval time parameters for the rest of the trains. Including $h_{aa}, h_{ap}, h_{pp}, h_{pd}, h_{pa}$
$c_k\left(i, i', t, t'\right)$	The cost of using space-time arc $\left(i, i', t, t'\right)$ for train *k*
m_l	The frequency of the train scheme line *l*
w_l	The first train with the earliest departure time is included in the train plan line
Q	The number of epochs in the main space-time network, which has $T' = Q \cdot T$ and $T'' = 2Q \cdot T$ for the given parameter *Q*
ϑ	The integer parameter is used to index each master plan in the expansion plan. $\vartheta \in \{0, \cdots, Q\}$
$q_{l,k}$	An integer parameter is used to specify the order in which the train *k* is in the train scheme line *l*. $q_{l,k} \in \{0, \cdots, m_l - 1\}$

Table 4: Variable definitions

Symbol	Definition
$x_k\left(i, i', t, t'\right)$	0-1 Main plan space-time arc selection variable, if train k select space-time arc $\left(i, i', t, t'\right)$, $x_k\left(i, i', t, t'\right) = 1$, or $x_k(i, i', t, t') = 0$
$y_k\left(i, i', \tau, \tau'\right)$	0-1 Extend plan space-time arc selection variable, if train k select space-time arc $\left(i, i', \tau, \tau'\right)$, $y_k\left(i, i', \tau, \tau'\right) = 1$, or $y_k(i, i', \tau, \tau') = 0$

describes the space-time resource occupancy constraints in the railway network based on the set train safety headway times. Specifically, only one train can occupy a space-time arc in the set $\phi(j, j', t'')$. Finally, Constraint specifies the type of space-time arc selection variables.

The set *A* denotes the collection of stations in a high-speed railway network, while the set *G* comprises all sections connecting pairs of adjacent stations. The periodic railway timetabling problem involves scheduling a set of train service lines $l \in L$ to repeat periodically with a period length *T*. For each train service line *l*, its operating frequency m_l within the period *T*, stopping pattern, and operational range are predefined. The frequency m_l requires that m_l identical trains be operated within period *T*, distributed uniformly at equal intervals. This uniformity constraint is termed the regularity requirement of periodic timetables.

Specifically, the time interval between the arrival or departure times of any two trains belonging to the same service line at identical stations must be an integer multiple of $\lfloor T/m_l \rfloor$. Zhang explored relaxing the regularity requirement to enhance scheduling flexibility from the perspective of railway line capacity analysis. However, this chapter strictly enforces the regularity requirement to facilitate subsequent modeling and solution procedures.

Additionally, the minimum and maximum running times of trains in sections and dwell times at stations are prescribed. The starting and stopping additional times of trains can be incorporated into the minimum and maximum section running times, provided the stopping patterns are predefined. However, accounting for these constraints may result in significant disparities in actual running times between trains traversing the same section. To prevent overtaking of slower trains by faster ones within a single section, this study adopts the approach of Lie, which involves splitting train running arcs in long sections by inserting a virtual station at the midpoint where overtaking might occur. Consequently, the starting and stopping additional times must be integrated into the minimum and maximum running times for the first and second virtual subsections, respectively.

In this model, block sections are treated as individual track circuits, each consisting of two nodes numbered sequentially from left to right (for clarity, this numbering does not follow the conventional switch numbering rules). An example is provided in 6.

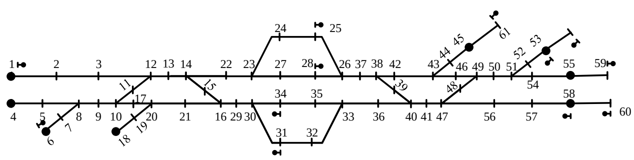


Figure 6: Numbered refined railway lines

The periodic train timetable problem essentially involves periodically scheduling a series of planned train space-time paths within each period of length *T*. Each path is associated with the operating frequency, stopping patterns, and running zones of the train. Additionally, to meet the periodic regularity requirements of trains, the arrival or departure times of any two trains with parallel paths in the space-time network must maintain a fixed time interval. Furthermore, the running time of trains on each track circuit (including dwell time if the train stops) is constrained within a specific time range. Under a given stopping pattern, trains can precisely control their acceleration and deceleration to manage time within the minimum and maximum running time limits. The railway line N is represented as a microscopic discrete space-time network graph as follows: the vertex set V consists of individual track circuits (or track circuit groups) in the railway network, and the directed arc set C represents the running lines through each track circuit. A railway operations department needs to schedule train timetables for three different physical routes. Since this study considers a microscopic-level railway network, both stations and sections can be represented by track circuits, which are further composed of nodes. Thus, the network can represent the entire railway system, with edge nodes representing train origin and destination nodes. Physical Route 1 is depicted by a blue dashed line, where the train departs from Node 1, briefly stops at track circuit (24, 25), and finally arrives at track circuit (45, 61) via Node 45. Physical Route 2 and Physical Route 3, represented by orange and red dashed lines, respectively, depart from Node 6 and Node 1, pass through the track circuits without stopping, and ultimately arrive at track circuit (56, 59) via Node 56, as shown in 7.

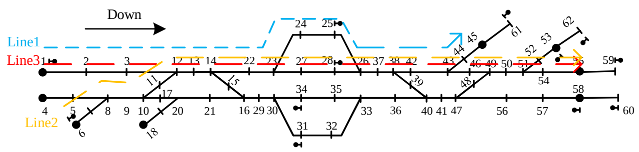


Figure 7: A railway line with edge nodes that contains three paths

In this model, the passage of a train through each track circuit involves three processes of the track section locking time: reservation process, running process, and release process. 8 illustrates the scenario where two consecutive trains from different service lines pass through two track circuits. In the space-time network, the directed arcs are connected end-to-end. Taking the black service line as an example, the two directed arcs represent the running process time of the train passing through track circuits, which is the time difference from when the train head enters the starting node of the track circuit to when the train tail leaves the ending node of the track circuit. This includes both the pure running time and the dwell time of the train.

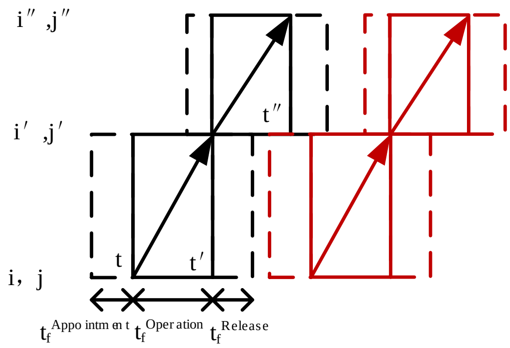
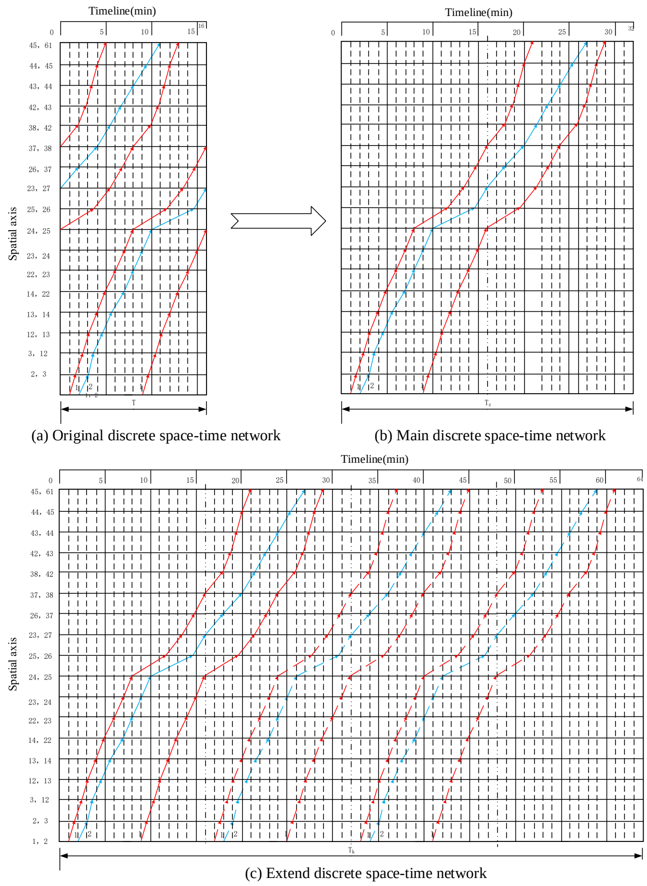


Figure 8: Directed arcs in discrete space-time networks



(a) Original discrete space-time network (b) Main discrete space-time network

(c) Extend discrete space-time network

Figure 9: Three discrete space-time networks

3.3 Model Assumptions

To facilitate processing and comprehension, several assumptions are made based on the scope and nature of the problem in this study:

1. The minimum time granularity of the space-time network model in this project is assumed to be 15 seconds. The time axis is divided into unit-length multiples, and a smaller minimum time granularity can be applied for higher precision.
2. The railway line in this problem is double-tracked. The model considers only trains in the down direction, assuming no conflicts with trains in the up direction.
3. All trains are assumed to depart from the initial peripheral nodes of the railway network and terminate at designated peripheral nodes.
4. Trains within the same service line are assumed to follow identical physical paths (i.e., traverse the same track circuit sections), with their space-time paths uniformly distributed within the period.

3.4 Periodic Timetabling Optimization Model

9 illustrates a periodic timetable with two distinct train service lines, both traversing Physical Path 1 on the railway line, thereby sharing the same track circuits by default. The period length T is 16 minutes, where Service Line 1 corresponds

to a train frequency of 2 (i.e., two trains depart within one period), and Service Line 2 corresponds to a frequency of 1. Additionally, trains make stops at track circuits (24, 25), with dwell time included in the total travel time. Within a single period, the temporal spans of the space-time paths for all three trains exceed the period boundary. Consequently, the original discrete space-time network cannot display the complete continuous service lines. Instead, portions of the space-time paths outside the current period are mapped into the period's space-time network, as shown in 9(a).

Next, we transform the original discrete space-time network into a master discrete space-time network (hereafter referred to as the 'master graph') to visualize the full continuous service lines. This is achieved by extending the timetable's temporal horizon to $H \cdot T$, forming a master space-time network with an expanded temporal span. Within this master network, space-time arcs of the same path across multiple periods are concatenated, retaining only one complete service line, as depicted in 9(b). The parameter H is set to the smallest integer satisfying $H \cdot T > T_{max}$, where T_{max} denotes the maximum possible travel time for any train to reach its destination. For example, in 9(b), T_{max}= 29 minutes. Thus, $H = [29/16] = 2$, resulting in a master temporal span T'= 32 minutes.

Finally, to reflect the periodicity of train service lines and ensure conflict-free periodic timetables, we further extend the temporal axis of the master graph to generate an extended discrete space-time network (hereafter the 'extended graph'). Assuming the extended period length is $T'' = H' \cdot T$, where $H' \geq H$, the extended graph replicates each service line H' times, with each replication shifted by T units along the temporal axis. As shown in 9(c), when H'= 2 and T''= 64 minutes, the service lines from the master graph are replicated twice, represented by dashed space-time arcs. In the extended graph, although space-time paths may cross the right boundary of the first period, the departure times of the first trains in all service lines must lie within the interval [0,T). Furthermore, the departure time window Δt for the first train in Service Line l is constrained to $\Delta t \subseteq [0, T/m_l)$, where m_l is the service frequency. According to the Lemma, subsequent trains in the same service line are uniformly distributed with equal intervals T/m_l, forming parallel directed arcs in the space-time network. For example, the first and second trains on Path 1 depart at 1 minute and 9 minute, respectively, with an interval of 16/2=8 minutes.

3.5 Model constraint

$$\min Z_1 = \sum_{k \in K} \sum_{(i,i',\tau,\tau') \in E'_k} c_k(i,i',\tau,\tau') \cdot y_k(i,i',\tau,\tau') \quad (4)$$

$$\sum_{i,t:(i,i',t,t') \in E_{w_l}} x_{w_l}(i,i',t,t') - \sum_{i,t:(i',i,t',t) \in E_{w_l}} x_{w_l}(i',i,t',t) = \begin{cases} -1 & i' = o_{w_l}, t' = dep_k^s \\ 1 & i' = d_{w_l}, t' = T' \\ 0 & \text{otherwise} \end{cases}, \quad \forall l \in L$$

$$y_k(i,i',\tau,\tau') = x_k(i,i',t+\vartheta T, t'+\vartheta T) \ \forall k \in K, (i,i',t,t') \in E, (i,i',\tau,\tau') \in E', \vartheta \in \{0,\cdots,Q\}, \tau = t+\vartheta T, \quad (5)$$

$$\tau' = t' + \vartheta T \quad (6)$$

$$\sum_{k \in K} \sum_{(i,i',\tau,\tau') \in \phi(j,j',\tau'')} y_k(i,i',\tau,\tau') \leq 1, \quad \forall (j,j') \in G, \tau'' \in T''$$

$$x_k(i,i',t,t') \in \{0,1\}, \ \forall k \in K, (i,i',t,t') \in E \quad (7)$$

$$y_k(i,i',t,t') \in \{0,1\}, \ \forall k \in K, (i,i',t,t') \in E \quad (8)$$

In terms of mathematical representation, space-time arc selection variables $x_k(i,i',t,t')$ and $y_k(i,i',t,t')$ are designed for the master plan and the extended plan, respectively. Specifically, the time units in the master space-time network are indexed by subscripts t and t', while the time units in the extended space-time network are indexed by subscripts τ and τ'. Equations 4 to 8 present the periodic train timetable optimization model based on the extended space-time network. In the model, the objective function 4 minimizes the total travel time of all trains in the extended plan. Constraint is the flow balance constraint, ensuring that the first train w_l in each train service line $l \in L$ can find a unique space-time path in the master space-time network. The variable $x_{w_l}(i,i',t,t')$ in constraint represents the space-time arc selection variable for the first train w_l in the master space-time network. Constraint generates the space-time paths for the remaining trains in the same train service line $l \in L$, excluding the first train w_l. These paths are obtained by shifting the space-time path of the first train w_l by integer multiples of the interval $min\{\lfloor T/m_l \rfloor, T-1\}$. Additionally, the integer parameter $q_{l,k}$, in constraint specifies the order of train k in train service line l, where $q_{l,k}$, belongs to the set $\{1,\cdot\cdot\cdot,m_l-1\}$, thereby excluding the first train w_l from constraint.

Another key contribution of the periodic train timetable optimization reconstruction model based on the extended space-time network is the use of variable separation and replication techniques. Constraint 6 is the consistency constraint between the master plan and the extended plan, which replicates the train space-time paths in the master plan Q+1 times to form the extended plan. Specifically, the integer parameter ϑ in constraint 6 controls the number of copies of the master plan required to form the extended plan, where ϑ belongs to the set $\{0,\cdot\cdot\cdot,Q\}$. The form of constraint 6 is similar to the nonanticipativity constraint in two-stage stochastic mixed-integer programming models that link the first and second-stage decisions. Crainic have demonstrated that the progressive hedging approach can effectively handle such constraints in two-stage stochastic mixed-integer programming models. Furthermore, since constraint 6 in the model only performs replication operations, there is no need to relax its dual into the objective function. Constraint **??** is the track capacity constraint, ensuring no train conflicts occur within the planning horizon of the extended space-time network, thereby guaranteeing the feasibility of the original periodic train timetable. Constraints 7 and

8 define the types of space-time arc selection variables in the master plan and the extended plan.

In addition to the aforementioned constraints, the representation of train timetables in discrete space-time networks inherently implies several fundamental constraints, including:
(1) Adjacent Track Circuit Arrival-Departure Temporal Constraint: The sequential connection of space-time arcs for the same train service ensures temporal continuity between adjacent track circuits. Specifically, the departure time from the preceding track circuit equals the arrival time at the subsequent track circuit.
(2) Space-Time Network and Physical Path Correspondence Constraint: The existence of a directed space-time arc in the discrete network explicitly indicates that the train selects track circuit (*i, j*) along its route from the origin to the destination node in the railway network, while the absence of such an arc implies non-selection.
(3) Running and Dwell Time Constraint: The traversal of track circuit (*i, j*) inherently incorporates both running time and dwell time. By default, the total time expenditure on the directed arc equals the temporal difference between the arrival time at the subsequent track circuit and the arrival time at the current track circuit. Consequently, these constraints are not explicitly formulated in the model construction.

4 Solution algorithm

4.1 Grouping by Path Similarity

The optimization of detailed operational timetable compilation requires modeling the railway network at a micro level. Although this approach allows for more efficient utilization of track conditions, it also significantly increases the complexity of the problem. To address this issue, reduce the complexity, and accelerate the algorithmic solution process, this paper proposes to establish methods for grouping trains.

Herrigel, while grouping passenger trains of the Swiss railway to solve the PESP model, proposed the geographical grouping (Geo) method based on the practical scheduling requirement of assigning railway staff to different regions to organize trains. Drawing on this idea, in the microscopic-level railway network, since the nodes included in the physical paths of different trains are not identical, we can group trains based on the similarity of the node sets they pass through. Cluster analysis, also known as classification analysis, is a statistical method that divides original objects into multiple relatively homogeneous groups, with the classes unknown prior to the analysis. Clustering partitions samples into several groups based on their distances or similarities, aiming to minimize intra-group distances while maximizing inter-group distances. Among clustering algorithms, K-Means is one of the most widely used. Since the input data is unlabeled, it belongs to unsupervised learning.

$$S\left(l_i, l_j\right) = 1 - \frac{N_{l_i \cap l_j}}{\left(N_{l_i} + N_{l_j}\right)/2}, \forall l \in L \tag{9}$$

Where the similarity distance $S \in [0,1]^{L \times L}$, L is a collection of all train routes, N_{l_i}, N_{l_j} are the number of nodes that path l_i, l_j passed.

Setting *k=m* involves dividing all trains into mm groups. A heuristic algorithm is designed to cluster and group the train paths, with the principle that the resulting groups should have approximately equal numbers of elements, and the elements within each group should minimize the total similarity with other elements in the same group. In other words, the most similar paths are assigned to the same group based on the similarity measure. The flowchart of the algorithm is shown in 10. The steps of this heuristic algorithm are as follows:
Definitions:

- Sample Set: $L = \{l_1, l_2, \ldots, l_n\}$
- Cluster Groups: $K = \{K_1, K_2, \ldots, K_m\}$

Procedure:
1.Initialization:

All cluster groups $K_j\,(j = 1, 2, \ldots, m)$ are initialized as empty sets.

2. Seed Selection:

- Select two elements $l_a, l_b \in L$ with the maximum pairwise dissimilarity (i.e., $\mathrm{argmax}_{l_i, l_j} d\left(l_i, l_j\right)$,where d denotes the distance metric).
- Assign l_a and l_b as initial centroids to distinct clusters K_1 and K_2 .
- Iteratively select the remaining $m-2$ elements from L to maximize the minimum inter-cluster dissimilarity,ensuring the sum of pairs similarities S is minimized.

Formally:

Select $l_k \in L \setminus \{K_1 \cup K_2\}$ such that $\sum\limits_{l_i \in K_j} S\left(l_k, l_i\right)$ is minimized for all j .

Assign these elements to new clusters $K_3, \ldots, K_m$.

3. Balanced Assignment:

- While $L \neq \varnothing$:

a. Identify the cluster $K_{\min}$ with the smallest cardinality.

b. Select an element $l \in L$ that minimizes the intra-cluster similarity when added to $K_{\min}$: $l^* = \mathrm{argmin}_{l \in L} \sum\limits_{l_i \in K_{\min}} S\left(l, l_i\right)$.

c. Assign l^* to $K_{\min}$ and remove l^* from L .

4.2 Sorting by Occupation Time

Since different groups occupy the railway line for varying total durations, prioritizing the solution of groups with longer occupation times can be beneficial. This is because their longer spans reduce the scale of the directed arc options in the space-time network for subsequent groups, thereby narrowing the search range during computational solving and accelerating the solution process.

The occupation time of a train group is the sum of the occupation times of all trains within the group, where the occupation time of a single train is the total time it occupies all track circuit sections it passes through, including both running and dwell times. Although the occupation times of different trains may overlap temporally, since the timetable is not yet

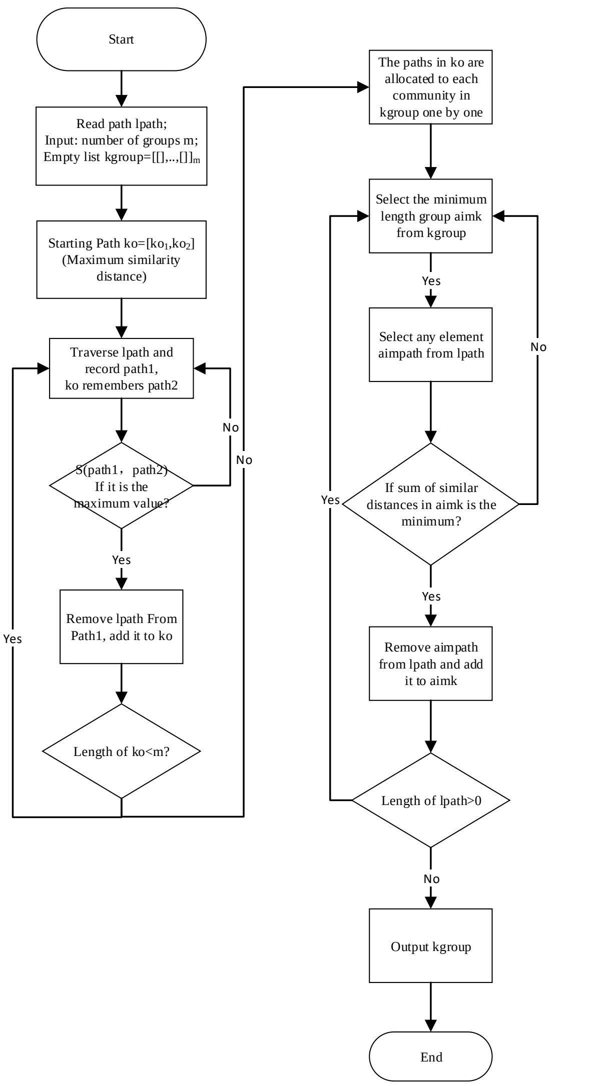


Figure 10: Flow diagram of path similarity grouping algorithm

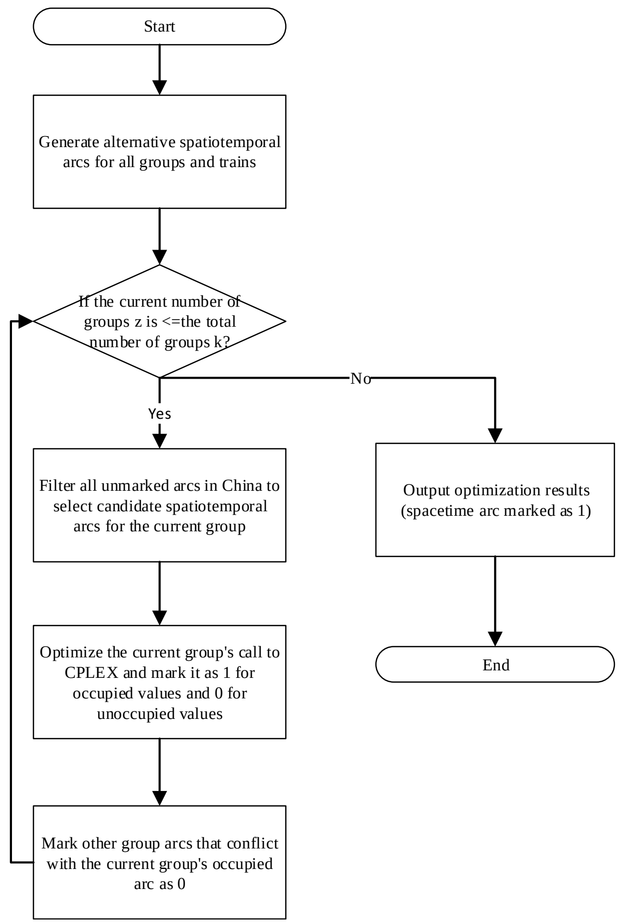


Figure 11: Flow diagram of the grouped iterative solution algorithm

determined when using this sorting method, the sum of occupation times is used as the sorting criterion. The formula for calculating the occupation time of a train group is as follows:

$$t_i = \sum_{a \in A_i} \sum_{link \in path_a} \left(\frac{L_{link}}{\alpha \cdot v_m^{link}} + t_{dw} \right) \tag{10}$$

Where the length of the track circuit segment L_{link} , the maximum allowable speed at which a train can pass through a section of the track circuit is v_m^{link} , the train stop times is t_{dw}, the velocity coefficient is α.

The flowchart of the algorithm is shown in 11. After completing the train grouping and sorting, it is essential to design a method for iteratively incorporating the grouped trains into the model for solving. After each target group optimization, the determined arcs are added to the 'Marked 1' list. During the next optimization, the arcs from the latest 'Marked 1' list, along with all arcs of the group to be optimized, are treated as the target optimization arcs. Constraints are added to the model to ensure that all arcs in the 'Marked 1' list are fixed to 1. After optimization, this list is updated accordingly.

5 Case study

This chapter adopts the network data from the 2016 RAS Problem Solving Competition (Railway Application Section of the Institute for Operations Research and the Management Sciences (INFORMS)) titled ‘Train Scheduling in Railway Networks: Integrated Optimization of Timetabling and Maintenance Task Allocation’. By configuring rational parameters and leveraging the mathematical models established earlier, this study solves the network to generate a refined periodic railway timetable for practical operations. The grouping algorithm, model-solving procedures, and timetable generation in this work are implemented using Python 3.7.12.

5.1 Case description and parameter configuration

The railway network provided in the 2016 RAS competition comprises 27 stations, 55 track segments, 261 routes, 1,027 track circuit section groups, 1,811 track circuit sections, and 1,619 nodes. The network is partitioned into five geographical divisions: Western, Eastern, Northern, Southern, and M-Station. The M-Station area represents the most complex hub, consisting of 19 siding tracks and 4 main tracks, interconnected with all four regional divisions. Consequently, most trains in the network traverse M-Station. The network also includes maintenance track circuit section groups, located in stations or track segments.

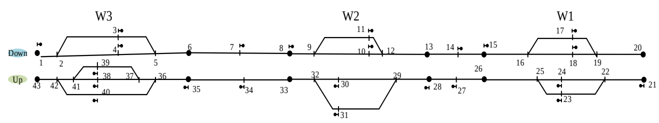


Figure 12: Two-track rail route map of the western region

Due to the excessively large number of track circuit sections in the entire railway network, the generated detailed timetable is inconvenient to display. This example extracts the western region of the network, focusing on three stations as the target line instance. To reduce the number of track circuit sections along the line, we combine several track circuit sections, as shown in 12. This regional line is a double-track railway comprising 43 nodes, 48 track circuit sections, and 3 stations (W1-W3). Since the model does not consider conflicts between trains in different directions, only the track circuit sections that may be passed by trains traveling in the downward direction from W3 to W1 are retained, totaling 14 sections. Additionally, as the mathematical model focuses solely on optimizing the train timetable, the original maintenance sections are not considered in this project.

The input data files include information on nodes, track circuit sections, track circuit section groups, and train details, with their specific meanings described in 5.

5.2 Parameter configuration

(1) Fixed Parameters

Following the methodology of Meng for setting occupation and release intervals of railway resources, the reservation time for track circuit sections is set to 60 seconds, meaning a track circuit section is occupied one minute before a train arrives. Similarly, the release time is set to 60 seconds, indicating the track circuit section remains occupied for one minute after the train departs.

In this case study, to avoid excessive computational complexity from a large number of space-time arcs, the dwell time at stations is fixed at 120 seconds, representing the minimum allowable dwell time. Additionally, the minimum traversal time for a train to pass a track circuit section is calculated based on the section length, maximum permitted speed, and train speed coefficient. The maximum traversal time is defined as 120% of the minimum traversal time, forming a candidate set of travel times for each train across track circuit sections. This approach effectively reduces the number of space-time arcs to be solved, particularly in networks with long or densely segmented tracks. The base period length of the periodic timetable is set to 2 hours (7200 seconds). Notably, the minimum time granularity is defined as 10 seconds, requiring all time-related variables in the case study to be converted into unit-scale quantities (e.g., the period duration is represented as 720 units). Finally, the master plan space-time network is extended to generate an extended plan space-time network, reflecting the regularity of train service lines. A minimum scaling factor of 2 is applied, fixing the extended period length as twice the master plan period length.

(2) Variable Parameters

After defining all fixed parameters, the period scaling factor (from the base plan to the master plan) is adjusted to finalize the periodic timetable configuration. The master plan period length is determined by ensuring that the latest arrival time of any service line (with its first train departing within the initial period) does not exceed the master plan period length.

The case study involves 8 trains, which can be grouped into 2, 3, or 4 clusters. The grouping principle prioritizes minimizing total computational time while ensuring that no single cluster contains an excessive number of trains, which would negate the purpose of the proposed grouping optimization framework.

5.3 Validation of train grouping methodology

To visually compare the performance of direct optimization versus group-based optimization, we designed two train path scales: a small-scale group (hereafter ‘Group 1’) with an average path length of 5 track circuit sections per train, and a large-scale group (hereafter ‘Group 2’) with an average path length of 10 track circuit sections per train. The selected track circuit regions for these groups are highlighted by the blue and red boxes in 13. Notably, each additional track circuit section doubles the number of space-time arcs generated per train. For example, a single train in Group 2 generates 31 times more space-time arcs than one in Group 1. For 6 trains, this difference escalates to 186 times.

The total number of trains in both groups was varied dynamically from 2 to 6. Using identical hardware, direct optimiza-

Table 5: The road network data for the case

Filename	Attribute	Meaning
Input_Node	node_id	Node number
Input_Link	link_id	Track circuit number
	from_node	Track circuit start node
	to_node	Track circuit end node
	length_in_mile	Track Circuit Line Length (miles)
	speed_limit_in_mph_FT	Pass the maximum speed limit (downlink)
	dwelling_allowable_flag	Whether trains are allowed to stop
Input_Cell	cell_id	The track circuit group number
	including_link	Included track circuits
Input_Train_Info	train_id	The train number
	origin_node_id	The starting point of the train
	destination_node_id	End of the train
	speed_multiplier	Velocity coefficient
	frequency	The frequency of the train during the original period
	link_of_ actual_path	The track circuit through which the physical path of the train passes in turn
	dwelling_link	Trains need to stop at the track circuit in turn

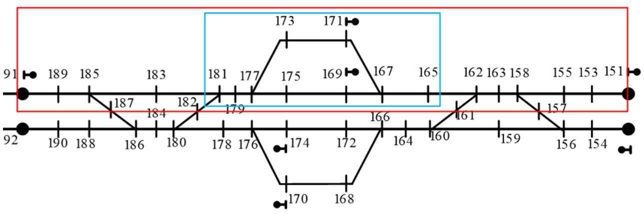


Figure 13: Track circuit section in the experimental area

tion and group-based optimization were applied to each experimental group. For group-based optimization, trains were randomly grouped, with a maximum of 2 trains per group to ensure computational tractability. The total computation times for model optimization in both experimental groups are compared in 14 and 15.

Key findings from the experiments include:

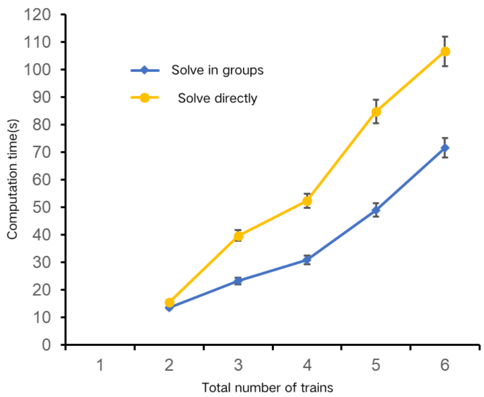


Figure 14: The first group of experiments computation time

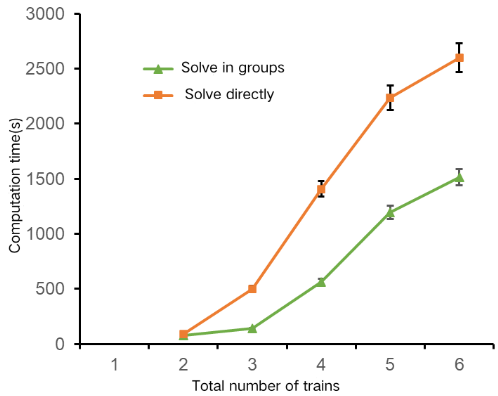


Figure 15: The second group of experiments computation time

1. Group-based optimization consistently reduced computation time compared to direct optimization, regardless of path scale.

2. As the total number of trains increased, computation time rose for both methods. However, the advantage of group-based optimization became more pronounced with larger fleets. For 6 trains, direct optimization exhibited significantly longer computation times, with the gap widening progressively.

3. For the same total number of trains, longer train paths (Group 2) amplified the time saving benefits of group-based optimization

4. Objective function values from both methods were nearly identical, confirming that group-based optimization preserves solution quality.

In summary, the results demonstrate that group-based optimization significantly reduces computation time for larger fleets, highlighting its practical superiority. Furthermore, the method exhibits even greater advantages when applied to large-scale networks with numerous track circuit sections and high train volumes.

5.4 Experimental design and results

To evaluate the effectiveness of different train grouping methods and group sequencing strategies, we conducted comparative experiments. The experimental case uses the track circuit sections within the blue-boxed region of 13, with a total of 12 trains.

(1) Comparison of Train Grouping Methods

Three grouping strategies were tested:

1. Random Grouping: Trains are randomly assigned to groups, with a maximum of 3 trains per group.

2. Path Similarity-Based Grouping: Trains sharing overlapping track circuit sections or similar physical paths are grouped together.

3. Departure-Time Clustering-Based Grouping: Trains are clustered using the K-means algorithm based on their scheduled departure times.

A total of 12 trains (representing 12 distinct service lines) were divided into 2 to 6 groups under three grouping strategies: route similarity-based grouping, frequency-based grouping, and random grouping. To control for extraneous variables, the sequencing of train groups was uniformly randomized across all configurations. Each configuration underwent three repeated trials, resulting in 45 total experiments. The objective function value remained consistent at 44,490 s in all trials. The averaged computation times are summarized in 6 and visualized in 16. The final result reveals that the route similarity-based grouping significantly outperformed both frequency-based grouping and random grouping in terms of average computation time, while the latter two methods exhibited comparable performance. All grouping methods preserved solution quality, without impacting the objective function value. Across all grouping strategies, average computation time increased proportionally with the number of groups, highlighting a trade-off between granularity and computational efficiency.

(2) Comparison of Train Group Sequencing Methods

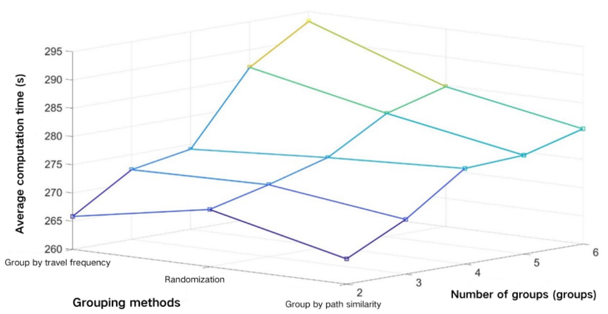


Figure 16: Chart of the results of the grouping method control experiment

The train grouping method of controlling irrelevant variables is divided into 6 groups by the method of route similarity. After the grouping is completed, the trainsets are sorted by random sorting and the total occupancy time. The total occupancy time of each group was calculated according to equation 10, and the experiment was repeated 3 times, and the sorting results and average calculation time are shown in 7.

The results show that the average calculation time is nearly 1.233% compared with random sorting, and it does not affect the objective function value, which proves that sorting by occupancy time from large to small can improve the computational efficiency of the model to a certain extent.

5.5 Solution of the practical case

In the example, the total length of the line is approximately 65 km. To simplify the calculation, only trains traveling in the downward direction are considered. As shown in 17, based on whether the trains stop at intermediate stations, there are two train operation paths (via nodes):

(1) With stops: $3 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17$

(2) Without stops: $3 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17$

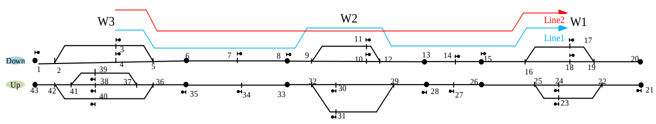


Figure 17: Actual train route map

The trains operate at a maximum speed of 180 km/h and are categorized into four types based on their stopping patterns at intermediate stations and line-specific conditions:

1. Type I: Non-stop at intermediate station W2 with a speed coefficient of 1;

2. Type II: Non-stop at intermediate station W2 with a speed coefficient of 0.7;

3. Type III: Stops at intermediate station W2 with a speed coefficient of 1;

4. Type IV: Stops at intermediate station W2 with a speed coefficient of 0.7.

Key operational characteristics of these train types, including acceleration/deceleration profiles, dwell times, and energy consumption metrics, are summarized in 8. According to the path similarity grouping method, combined with the train operating frequency, all trains are divided into 4 groups. The train grouping and solving order are shown in 9. 10 shows the detailed timetable for each train that generates the first train entering the track circuit.

Using Python's Matplotlib library, a microscopic-level periodic timetable was generated, as illustrated in 18. Finally, following the methodology of Andrea for generating train block time diagrams, we constructed periodic timetables by filling closed rectangular blocks (aligned with diagonal space-time arcs) with track circuit traversal times or track circuit blocking times. The resulting diagrams are shown in 19 and 20, respectively.

Table 6: Comparison table of experimental results of grouping methods

Groups' number \ Method	Path similarity	Travel frequency	Randomization	Average
2	264.493	270.158	265.852	266.834
3	269.686	272.753	272.336	271.592
4	276.930	275.772	274.185	275.629
5	277.488	281.824	286.893	282.068
6	280.373	284.768	293.256	286.132
Average	273.794	277.055	278.504	–

Table 7: Experiment of trainset sequencing method

	(1) 1,6	(2) 7,12	(3) 2,8	(4) 3,9	(5) 4,10	(6) 5,11	Average calculation time
Sort randomly	3	2	5	6	4	1	280.373
Occupy time sorting	4	3	1	6	2	5	276.917

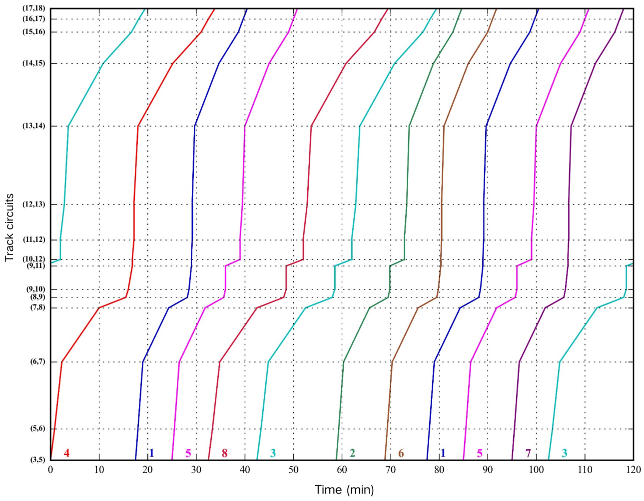


Figure 18: Periodic train timetable

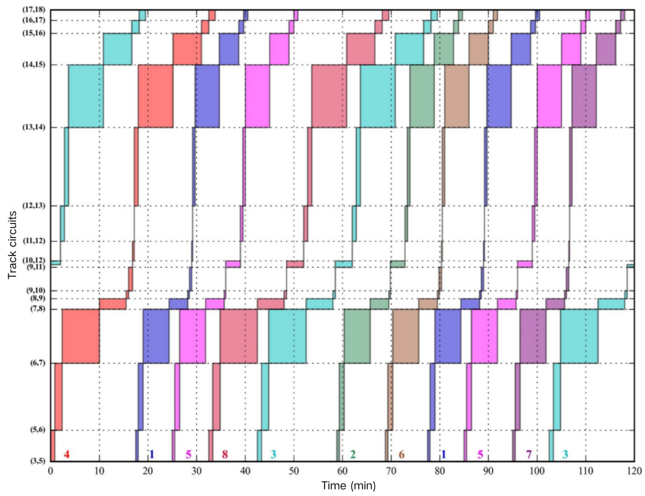


Figure 19: Periodic train timetable (Track circuit section running time)

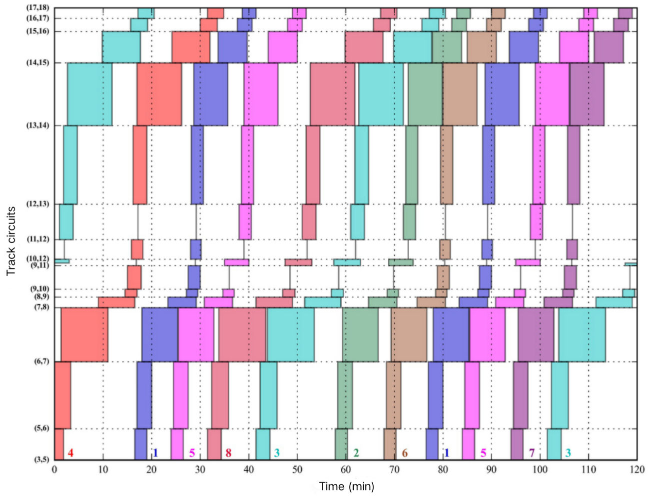


Figure 20: Periodic train timetable (Track circuit section locking time)

6 Conclusion

At the microscopic level, railway stations and track sections are modeled with train movements represented through track circuit units along physical paths. Train occupation of these track circuits is regulated based on locking times to ensure safe operation. By representing time on the horizontal axis and traversed track circuits on the vertical axis, a sequence of space-time network diagrams is constructed: the original, the main, and the extended space-time networks. The mathematical model is formulated using the arcs of the extended space-time network, aiming to minimize total travel time subject to constraints including flow conservation, periodicity coupling, track circuit occupancy, and binary decision variables. Due to its combinatorial nature, the resulting integer programming model is NP-hard. To alleviate computational complexity and enhance solution efficiency, this study proposes various train grouping strategies and ordering heuristics, alongside a train-group iterative solution algorithm. The model is implemented

Table 8: Train information

The train number	Velocity coefficient	Frequency	Pass through the track circuit section	Stop track circuit section
1	1	2	1;2;3;4;5;6;8;10;11;12;13;14	
2	1	1	1;2;3;4;5;7;9;10;11;12;13;14	7
3	0.7	2	1;2;3;4;5;7;9;10;11;12;13;14	7
4	0.7	1	1;2;3;4;5;6;8;10;11;12;13;14	
5	1	2	1;2;3;4;5;7;9;10;11;12;13;14	7
6	1	1	1;2;3;4;5;6;8;10;11;12;13;14	
7	1	1	1;2;3;4;5;6;8;10;11;12;13;14	
8	0.7	1	1;2;3;4;5;7;9;10;11;12;13;14	7

Table 9: Train group information

Group number	The number of the train in the group	Solve order
1	1,6	3
2	2,8	2
3	3,5	1
4	4,7	4

in Python with IBM ILOG CPLEX as the solver.

Model validation employs real-world data from the 2016 RAS problem-solving competition. Eight trains are partitioned into four groups using a path similarity clustering method, which are then optimized sequentially based on descending total occupation times. The grouped approach reduces computation time to 2710.894 seconds, yielding a 28.49% efficiency gain compared to ungrouped solving. Within a 2-hour scheduling horizon, the total travel time for down-direction trains reaches 18860 seconds (5.24 hours), demonstrating effective utilization of line capacity and improved timetable quality.

The optimization of train timetables at such a fine-grained microscopic level not only enhances operational efficiency but also directly benefits daily life by improving punctuality and reliability of rail services. This leads to reduced passenger waiting times, smoother transfers, and increased capacity to accommodate growing travel demand. Consequently, the proposed approach contributes to more sustainable and user-friendly public transportation systems, promoting economic development and enhancing the overall quality of urban mobility.

Virtual Stations and Intelligent Decision-Making: Intermediate stations are virtualized into two stations—one for ”arrival” and one for ”departure.” An AI system can intelligently decide on train stop or through-running strategies at these virtual stations based on real-time passenger flow, line conditions, and train priorities.

Train Paths and Predictive Analytics: Train paths include track circuits and switches. For safety, only one train can occupy a path at any given time. AI models can analyze historical data to predict potential conflict points for different trains on specific paths and perform proactive scheduling optimization.

Train Path Blocking Time and Intelligent Optimization: This includes reservation time (for signal and route setting, driver reaction time, and approach time to the entry signal), running time (from the train's front end entering to its front end exiting the path, calculated by summing running times on track circuits), and release time (clearance time for train length plus route unlocking time). An AI system can dynamically adjust these parameters to adapt to varying operational conditions, such as adverse weather or equipment failures, thereby achieving more flexible and efficient scheduling.

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Table 10: Detailed train schedules

Track circuits	Length (km)	Speed limit (km/h)	Entry time (minutes:seconds)							
			1	2	3	4	5	6	7	8
(3,5)	0.3	40	17:30	58:50	42:30	00:00	25:00	68:50	95:00	32:30
(5,6)	1.2	80	18:00	59:20	43:20	00:50	25:30	69:20	95:30	33:20
(6,7)	12.8	150	19:00	60:20	44:50	02:20	26:30	70:20	96:30	34:50
(7,8)	10.35	180	24:20	65:40	52:30	10:00	31:50	75:40	101:50	42:30
(8,9)	0.4	80	28:10	69:30	58:00	15:30	35:40	79:30	105:40	48:00
(9,10)	0.3	40	28:30	——	——	16:00	——	79:50	106:00	——
(9,11)	0.9	60	——	69:50	58:30	——	36:00	——	——	48:30
(10,12)	0.25	100	29:00	——	——	16:50	——	80:20	106:30	——
(11,12)	0.75	100	——	72:50	62:00	——	39:00	——	——	52:00
(12,13)	1.35	180	29:10	73:20	62:50	17:10	39:30	80:30	106:40	52:50
(13,14)	15	180	29:40	73:50	63:40	18:00	40:00	81:00	107:10	53:40
(14,15)	12	180	34:40	78:50	70:50	25:10	45:00	86:00	112:10	60:50
(15,16)	1.2	80	38:40	82:50	76:40	31:00	49:00	90:00	116:10	66:40
(16,17)	0.5	40	39:40	83:50	78:10	32:30	50:00	91:00	117:10	68:10
(17,19)	0.4	60	40:30	84:40	79:30	33:50	50:50	91:50	118:00	69:30

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